# Guidance - Ball on Rod Test Geometry and Hertzian Calculations

This table shows the relationship between the load on the contact, the corresponding axial load on the machine and the hertzian contact pressure. So, for example, an axial load set on the machine of 310.70 N, will result in an axial load per ball 103.57 N (there are three balls reacting the axial load), which will result in a radial load between ball and rod of 500 N.

**12.7 mm diameter steel ball on 9.525 mm diameter steel rod**

**12.7 mm diameter silicon nitride ball on 9.525 mm diameter steel rod**

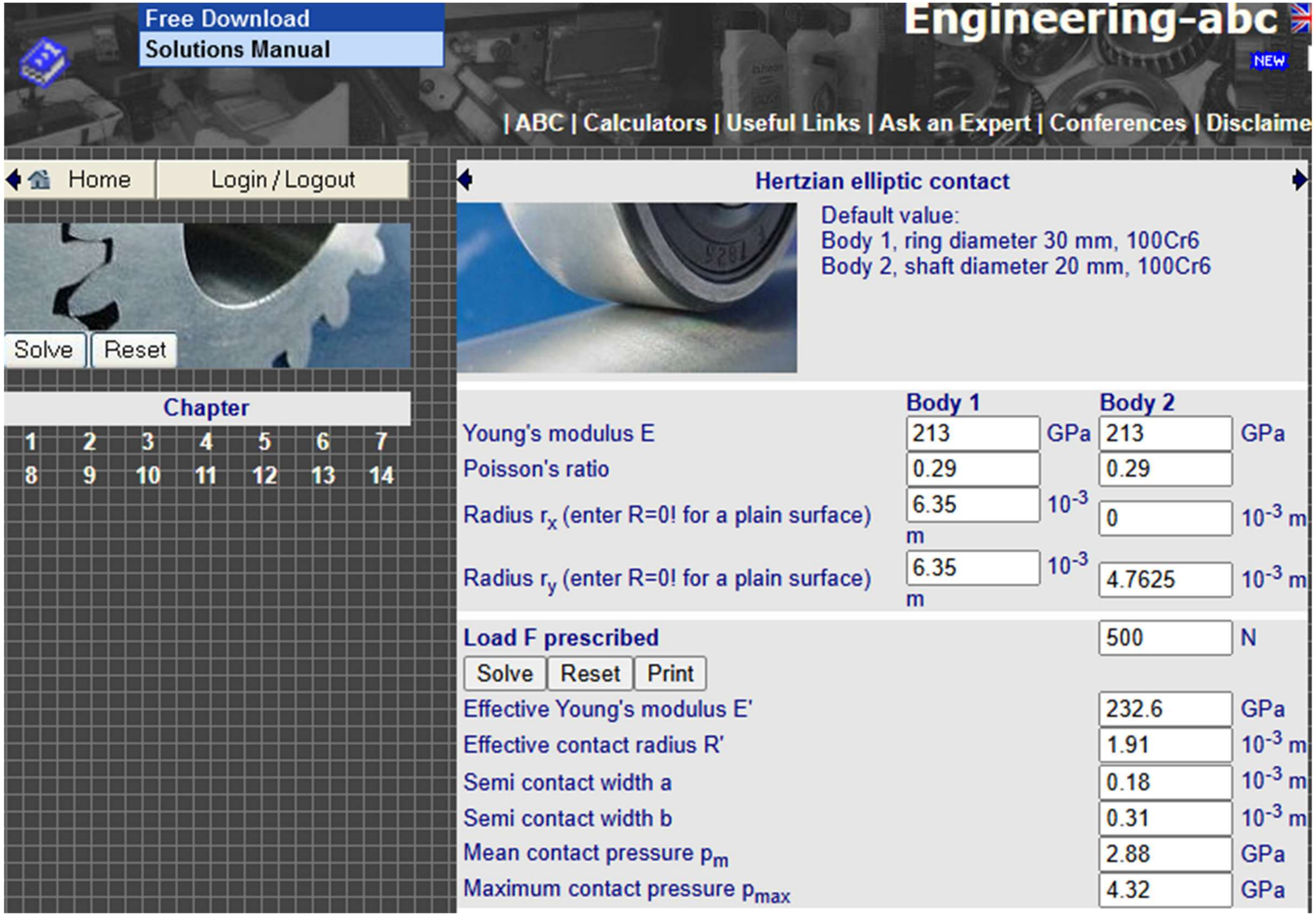
| Contact Load | Angle | Radians | Tan | Axial Load per Ball | Total Axial Load | Steel | Silicon Nitride |
|---|---|---|---|---|---|---|---|
| 100.00 | 22.5 | 0.3928 | 0.4143 | 20.71 | 62.14 | 2.52 | 2.81 |
| 200.00 | 22.5 | 0.3928 | 0.4143 | 41.43 | 124.28 | 3.18 | 3.53 |
| 300.00 | 22.5 | 0.3928 | 0.4143 | 62.14 | 186.42 | 3.64 | 4.05 |
| 400.00 | 22.5 | 0.3928 | 0.4143 | 82.85 | 248.56 | 4.01 | 4.45 |
| 500.00 | 22.5 | 0.3928 | 0.4143 | 103.57 | 310.70 | 4.32 | 4.80 |
| 600.00 | 22.5 | 0.3928 | 0.4143 | 124.28 | 372.85 | 4.59 | 5.10 |
| 700.00 | 22.5 | 0.3928 | 0.4143 | 145.00 | 434.99 | 4.83 | 5.37 |
| 800.00 | 22.5 | 0.3928 | 0.4143 | 165.71 | 497.13 | 5.05 | 5.61 |
| 900.00 | 22.5 | 0.3928 | 0.4143 | 186.42 | 559.27 | 5.25 | 5.84 |
| 1000.00 | 22.5 | 0.3928 | 0.4143 | 207.14 | 621.41 | 5.44 | 6.04 |
| 1100.00 | 22.5 | 0.3928 | 0.4143 | 227.85 | 683.55 | 5.61 | 6.24 |
| 1200.00 | 22.5 | 0.3928 | 0.4143 | 248.56 | 745.69 | 5.78 | 6.42 |

So, we then need the hertzian contact equation for a ball on a rod. In this case we will enter the radius of the ball and the radius of the rod, plus a load of 500 N, plus the Poisson's Ratio and elastic modulus for steel on steel:

| R1x | Sigma 1 | E1 | R2x | Sigma 2 | E2 |
|---|---|---|---|---|---|
| m | | Pa | m | | Pa |
| | | | | | |
| 0.00635 | 0.29 | 2.13E+11 | 0.0047625 | 0.29 | 2.13E+11 |
| R1y | | | | | |
| m | | | | | |
| 0.00635 | | | | | |
| | | | | | |
| **Derived:** | | | | | |
| | | | | | |
| 1/Rx* | 0.37 | | Rx: Major Axis | | |
| | | | | | |
| 1/Ry* | 0.16 | | | | |
| | | | | | |
| 1/R* | 0.52 | | | | |
| | | | | | |
| R* | 0.0019 | m | | | |
| | | | | | |
| λ | **2.33E+00** | | | | |
| λ | **4.29E-01** | | | | |
| | | | | | |
| 1/E* | 8.60E-12 | | | | |
| | | | | | |
| E* | 1.16279E+11 | Pa | Contact Modulus | | |
| | | | | | |
| E' | 2.32558E+11 | Pa | Reduced Elastic Modulus | | |
| | | | | | |
| **Smallest λ:** | | | | | |
| | | | | | |
| λ | 4.29E-01 | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| **Calculate k:** | | | | | | |
| 1/k | **1.74E+00** | | | | | |
| | | | | | | |
| k | **5.74E-01** | | | | | |
| | | | | | | |
| a*^3 | 4.60E-01 | | | | | |
| | | | | | | |
| a* | 7.72E-01 | | | | | |
| | | | | | | |
| b* | 1.35E+00 | | | | | |
| | | | | | | |
| b*^3 | 2.43E+00 | | | | | |
| | | | | | | |
| **Calculate:** | | | | | | |
| | | | | | | |
| **W** | **500** | **N** | **0.5** | **kN** | | |
| | | | | | | |
| a^3 | 5.66E-12 | | | | | |
| | | | | | | |
| **a** | 1.78E-04 | **m** | **0.178** | **mm** | | |
| | | | | | | |
| b^3 | 2.99E-11 | | | | | |
| | | | | | | |
| **b** | 3.11E-04 | **m** | **0.311** | **mm** | | |
| | | | | | | |
| **Pmean** | 2.87E+09 | **Pa** | **2.87** | **GPa** | | |
| | | | | | | |
| **Pmax** | 4.31E+09 | **Pa** | **4.31** | **GPa** | | |
| | | | | | | |

This then gives a value of contact pressure of 4.31 GPa for a contact load of 500 N. OK, we have entered 4.32 GPa as the result in our table, but that will probably be because, when doing the calculation, we may have used a slightly different value for Young's Modulus or Poisson's Ratio. Alternatively, we may have used an on-line calculator, which used a slightly different number of decimal places in its calculation, for example, here is the result we get using the on-line Tribology-ABC calculator:



In this case, we get a contact pressure of 4.32 GPa, so within 0.2% of our in-house calculation.

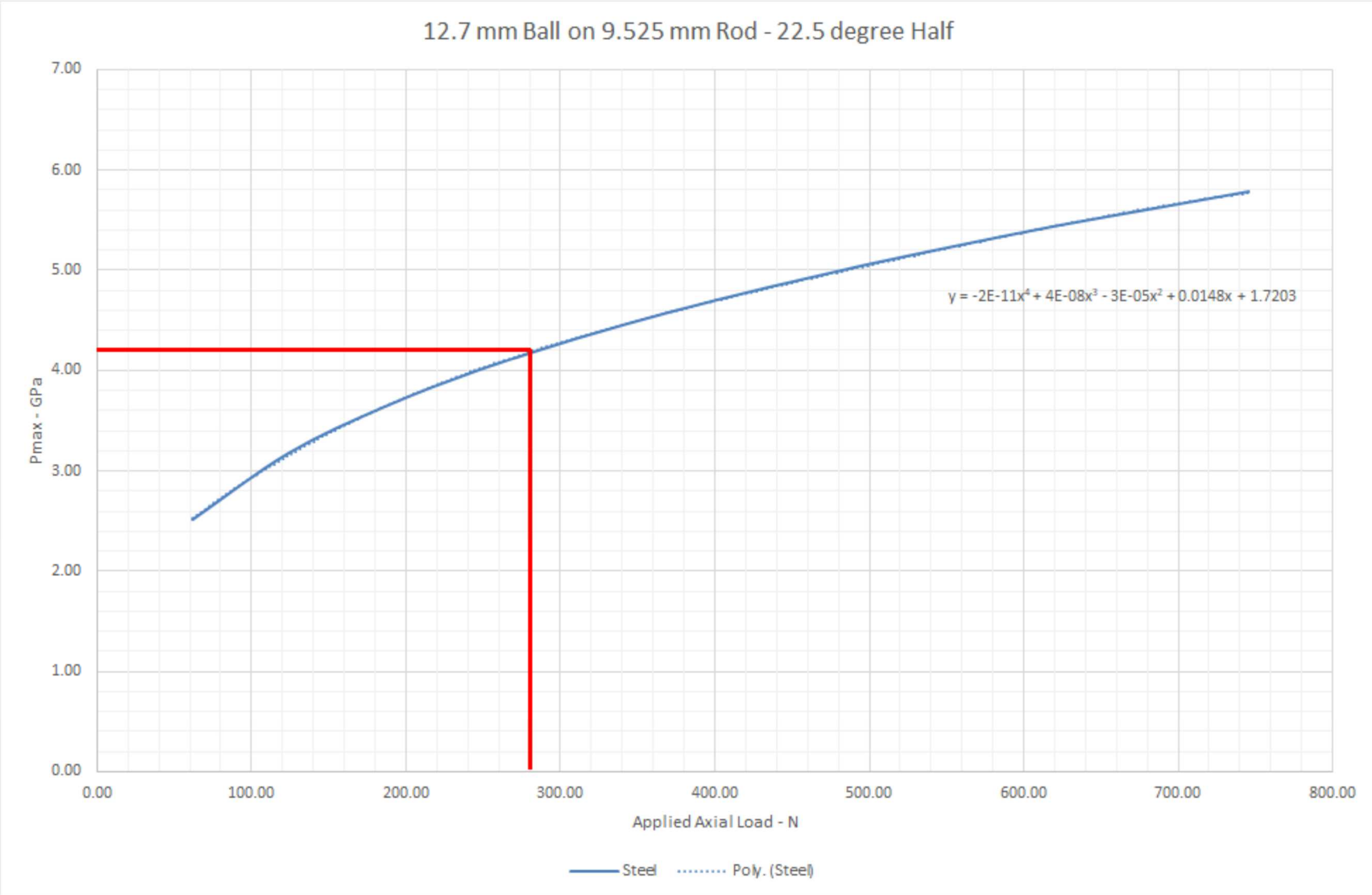
I am guessing that we used the on-line calculator to populate the table for the two different materials.

We then used the Hertzian contact equations to plot the graphs of hertzian contact pressure against load and to derive the equations for the curves. We then use these to calculate the load needed to generate a given contact pressure. Let’s say we want a hertzian contact pressure of, say, 4.2 GPa; what axial load do we have to apply?

We can see from our table of contact pressures, that to the Total Axial Load will have to be somewhere between 248.56 N and 310.70 N:

| Contact Load | Angle | Radians | Tan | Axial Load per Ball | Total Axial Load | Steel |
|---|---|---|---|---|---|---|
| 400.00 | 22.5 | 0.3928 | 0.4143 | 82.85 | 248.56 | 4.01 |
| 500.00 | 22.5 | 0.3928 | 0.4143 | 103.57 | 310.70 | 4.32 |

And from the graph, it looks as if the axial load should be about 280 N:


12.7 mm Ball on 9.525 mm Rod - 22.5 degree Half
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
Pmax - GPa
y = -2E-11x⁴ + 4E-08x³ - 3E-05x² + 0.0148x + 1.7203
0.00
100.00
200.00
300.00
400.00
500.00
600.00
700.00
800.00
Applied Axial Load - N
Steel
Poly. (Steel)

If we don't want to take the value from the graph, we could work out the exact load using the equation from the curve, in this example:

$$y = -2E-11x4 + 4E-08x3 - 3E-05x2 + 0.0148x + 1.7203$$

$$\text{Contact Pressure} = -2E-11(\text{Axial Load})^4 + 4E-08(\text{Axial Load})^3 - 3E-05(\text{Axial Load})^2 + 0.0148(\text{Axial Load}) + 1.7203$$

But this is not exactly a fun thing to do! A simpler solution is to use our geometric calculator to determine the radial load from a given axial load. We do this by adjusting the Contact Load value until we get a Total Axial Load Value of 280 N.

| Contact Load | Angle | Radians | Tan | Axial Load per Ball | Total Axial Load | Steel |
|---|---|---|---|---|---|---|
| 400.00 | 22.5 | 0.3928 | 0.4143 | 82.85 | 248.56 | 4.01 |
| 450.50 | 22.5 | 0.3928 | 0.4143 | 93.32 | 279.95 | |
| 500.00 | 22.5 | 0.3928 | 0.4143 | 103.57 | 310.70 | 4.32 |

The graph suggests that for an axial load of 280 N, we should get a contact pressure of 4.2 GPa. The table suggests that an axial load should result in a contact load of 450.50 N. Putting this into the Tribology-ABC calculator, we 450.0 N gives us 4.17 GPa, so not far of our predicted 4.2 GPa.

If we use the reverse calculation provided in the calculator, it indicates that the contact load should be 460.7 N for a contact pressure of 4.2 GPa:

## Hertzian elliptic contact

Default value:
Body 1, ring diameter 30 mm, 100Cr6
Body 2, shaft diameter 20 mm, 100Cr6

| | Body 1 | | Body 2 | |
|---|---|---|---|---|
| Young's modulus E | 213 | GPa | 213 | GPa |
| Poisson's ratio | 0.29 | | 0.29 | |
| Radius $r_x$ (enter R=0! for a plain surface) | 6.35 | $10^{-3}$ m | 0 | $10^{-3}$ m |
| Radius $r_y$ (enter R=0! for a plain surface) | 6.35 | $10^{-3}$ m | 4.7625 | $10^{-3}$ m |

**Load F prescribed** 450.5 N

Solve Reset Print

| | | |
|---|---|---|
| Effective Young's modulus E' | 232.6 | GPa |
| Effective contact radius R' | 1.91 | $10^{-3}$ m |
| Semi contact width a | 0.17 | $10^{-3}$ m |
| Semi contact width b | 0.3 | $10^{-3}$ m |
| Mean contact pressure $p_m$ | 2.78 | GPa |
| Maximum contact pressure $p_{max}$ | 4.17 | GPa |

**Contact pressure pmax prescribed** ($p_{max}$=1.5 $p_m$) 4.2 GPa

Solve Reset Print

| | | |
|---|---|---|
| Effective Young's modulus E' | 232.6 | GPa |
| Effective contact radius R' | 1.91 | $10^{-3}$ m |
| Semi contact width a | 0.17 | $10^{-3}$ m |
| Semi contact width b | 0.3 | $10^{-3}$ m |
| Mean contact pressure $p_m$ | 2.8 | GPa |
| Contact load F | 460.7 | N |

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If we put 460.70 N into our geometric calculator, it indicates that our applied test load should actually be 286.28 N as opposed to our guess of 280 N.

However, we are talking about a difference between 280 and 286 N of just 0.03 GPa, or a difference of 0.7%; this is not material in the context of rolling contact fatigue life.

| Contact Load | Angle | Radians | Tan | Axial Load per Ball | Total Axial Load | Steel |
|---|---|---|---|---|---|---|
| 400.00 | 22.5 | 0.3928 | 0.4143 | 82.85 | 248.56 | 4.01 |
| 460.70 | 22.5 | 0.3928 | 0.4143 | 95.43 | 286.28 | |
| 500.00 | 22.5 | 0.3928 | 0.4143 | 103.57 | 310.70 | 4.32 |

You should note that in most cases contact pressure is given to no more than one decimal place; any axial load from 276 N to 295 N, will give a contact pressure to one decimal place of 4.2 GPa. You should note that the control resolution of the test machines is to +/- 1 N.

We should perhaps resist the temptation to attribute spurious precision to hertzian contact equations and too many decimal places in related calculations.

**It is more than accurate enough to estimate the axial load needed to give a required contact pressure from the Contact Pressure – Axial Load Graphs.**

| Contact Load | Angle | Axial Load per Ball | Total Axial Load | Steel |
|---|---|---|---|---|
| 400 | 22.5 | 83 | 249 | 4.0 |
| 461 | 22.5 | 95 | 286 | 4.2 |
| 500 | 22.5 | 104 | 311 | 4.3 |

And finally, bear in mind that the contact pressure changes immediately the test starts, because in seconds the wear track will cease to be 9.525 mm or 10 mm diameter.